$$M_\infty = 2.796C_1/(p_{t,2}/p_{t,1})^{0.335} \text{ for } p_{t,2}/p_{t,1} < 2.5 \times 10^{-3} \quad (2b)$$

$$p_\infty = (15.5885 p_{t,1}C_2)/(M_\infty^2 + 3)^{2.5} \quad (3)$$

$$T_\infty = (3T_{t,1}C_3)/M_\infty^2 + 3 \quad (4)$$

$$q_\infty = 0.833 p_\infty M_\infty^2 \quad (5)$$

$$A^*/A = (16M_\infty C_4)/(M_\infty^2 + 3)^2 \quad (6)$$

where, from Ref. 5, the correction factors are

$$C_1 = 1 + p_{t,1}[(0.5114/T_{t,1}^{1.3744}) - (1.1937 \times 10^2/T_{t,1}^{2.5901})] = 1 + 1.5833 \times 10^{-4}p_{t,1}$$

$$C_2 = 1 + p_{t,1}[(1.4538/T_{t,1}^{1.3640}) - (1.3801 \times 10^3/T_{t,1}^{2.8233})] = 1 + 4.7498 \times 10^{-4}p_{t,1}$$

$$C_3 = 1 + p_{t,1}[(0.7901/T_{t,1}^{1.3628}) - (2.4311 \times 10^1/T_{t,1}^{2.1286})] = 1 + 2.0581 \times 10^{-4}p_{t,1}$$

$$C_4 = 1 + p_{t,1}[(1.7301/T_{t,1}^{1.3776}) - (5.3966 \times 10^2/T_{t,1}^{2.6027})] = 1 + 4.8359 \times 10^{-4}p_{t,1}$$

[The second form of these correction factors applies for ambient reservoir temperature ($T_{t,1} = 295°K$)]. The correction factors $C_2$ and $C_3$ are accurate to within 0.2% for Mach numbers above 20, but $C_2$ may be as much as 1% too large and $C_3$ 0.5% too large for Mach numbers between 10 and 20. The correction factor $C_4$, although not presented in Ref. 5, was obtained at the same time as the other factors and has an uncertainty of about 1.5% for all conditions. (The correction factor $C_1$ is independent of Mach number.) For ideal-helium, $C_1 = C_2 = C_3 = C_4 = 1$. The freestream Reynolds number, per foot, is given by

$$Re_\infty = 8.7479 \times 10^3(M_\infty p_\infty/T_\infty^{1/2}\mu_\infty) \quad (7)$$

where $\mu_\infty$ is found from Eq. (1). The system of units for the equations in this note is given in the nomenclature.

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# Response of a Circular Elastic Shell to Moving and Simultaneous Loads


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## Introduction

MUCH of the existing experimental information on the response of impulsively loaded, re-entry vehicle-type structures has been obtained by testing with sheet explosives; e.g., see Ref. 1. For many applications the goal of an impulse simulation is to impart a simultaneously applied, short duration, pressure pulse to the structure. However, sheet explosives impart an impulse by means of a short duration, pressure pulse which travels at the detonation velocity of the explosive. Thus, rather than providing the desired simultaneously applied impulsive load, the structure is loaded by a traveling, short duration, pressure pulse. The accuracy of simulating simultaneously applied impulsive loads with sheet explosives has recently been investigated by Florence[2] for beams and by the authors[3] for ring structures. In these papers, a comparison is made between the response produced by traveling forces which represent detonation waves in the sheet explosive and the response produced by simultaneously applied impulsive loads. Results presented in Refs. 2 and 3 can be used to evaluate the use of sheet explosives to simulate simultaneously applied impulsive loads.

In Ref. 3, a formula for the membrane stress in a long, circular, elastic shell produced by two identical forces, traveling from $\theta = 0$ to $\theta = \pm\pi/2$, is derived. Numerical results are presented for the membrane stresses produced by simultaneous, impulsive loads applied over one-half the shell circumference and two concentrated forces that move from $\theta = 0$ to $\theta = \pm\pi/2$ at a constant velocity $V$. It is shown in Fig. 4 of Ref. 3 that both loadings produce nearly the same response for a cosine distributed impulse and forces whose magnitude vary as the cosine function. The explosive loading technique modeled in Ref. 3 consists of wrapping variable thickness, sheet explosive one-half way around the structure and detonating the explosive with an exploding bridgewire which extends down the length of the cylinder.

In this Note, the accuracy of a similar impulse simulation technique for structures with circular cross sections is evaluated. Briefly, this method consists of wrapping sheet explosive one-half way around the structure and detonating the explosive at $\theta = -\pi/2$. Then the structure is loaded by a short duration, pressure pulse which travels from $\theta = -\pi/2$ to $\theta = +\pi/2$ at a constant velocity $V$. A detailed explanation of this method is presented by Lindberg in Ref. 4.

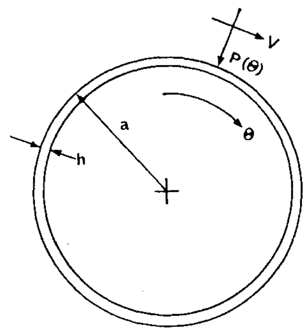


Fig. 1 Geometry of the problem.

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### Analysis

A wave equation for the membrane stress in a thin, infinitely long, circular, elastic shell subjected to a transient surface pressure is presented in Ref. 3. The circumferential membrane stress $\sigma$ is governed by

$$\sigma'' - \sigma - \ddot{\sigma} = (a/h)p(\theta,\tau) \tag{1}$$

in which primes denote $\theta$ derivatives and dots denote differentiation with respect to the dimensionless time variable $\tau$ given by

$$\tau = ct/a,\ c = [E/\rho(1 - \nu^2)]^{1/2} \tag{2}$$

The angular position $\theta$, the shell radius $a$, and the shell thickness $h$ are shown in Fig. 1; the applied pressure $p$ is measured positive radially inward; $E$, $\nu$, and $\rho$ are Young's modulus, Poisson's ratio, and density; $c$ is the speed at which disturbances propagate in the shell; and $t$ is time.

The membrane stress is obtained by following the procedure outlined in Ref. 3. A solution for $\sigma$ is taken in the form

$$\sigma(\theta,\tau) = \sum_{n=-\infty}^{+\infty} \psi(\theta + 2n\pi,\tau) \tag{3}$$

This form of solution extends the range of $\theta$ to the interval $-\infty < \theta < +\infty$ and permits the use of the Fourier integral over the angular variable $\theta$.

For a circumferentially moving line load traveling from $\theta = -\pi/2$ to $\theta = \pi/2$ which varies as $P(\theta)$ and travels at a constant velocity $V$, Eq. (1) becomes

$$\psi'' - \psi - \ddot{\psi} = \sigma_1 f(\theta)\delta(\tau - \alpha\theta) \tag{4a}$$

$$f(\theta) = \begin{cases} P(\theta)/P_0 & \text{for } |\theta| \leq \pi/2 \\ 0 & \text{for } |\theta| > \pi/2 \end{cases} \tag{4b}$$

$$\sigma_1 = \alpha P_0/h,\ \alpha = c/V \tag{4c}$$

where $P(\theta)$ is the magnitude of the line load, $P_0$ is a characteristic magnitude, and $\delta(\tau)$ is the Dirac delta function. The solution for $\psi$ is obtained by employing the method outlined in Ref. 3, and

$$\frac{\psi(\theta,\tau)}{\sigma_1} = \frac{-1}{2}\int_0^{\pi} f(\eta)H[\tau - \alpha\eta - |\theta - \eta|]J_0\{[(\tau - \alpha\eta)^2 - (\eta - \theta)^2]^{1/2}\}d\eta \tag{5}$$

where $H(\tau)$ is the Heaviside function and $J_0$ is a Bessel function of the first kind of order zero. The membrane stress can then be calculated from Eqs. (3) and (5) when $f(\theta)$ is specified.

### Discussion and Numerical Results

The accuracy of the impulse simulation technique described in Ref. 4 can be examined from the data presented in Fig. 2. The dashed line in Fig. 2 is the response at $\theta = 0$ produced by a simultaneously applied, impulsive load whose magnitude is $I \cos\theta$ over $|\theta| < \pi/2$ and zero over $\pi/2 < |\theta| < \pi$; whereas the solid line is the response produced by a force moving from $\theta = -\pi/2$ to $\theta = +\pi/2$ for $\alpha = 0.73$ and $P(\theta) = P_0 \sin(\pi/2 + \theta)$ shifted in time by the factor $\alpha\pi/4$.

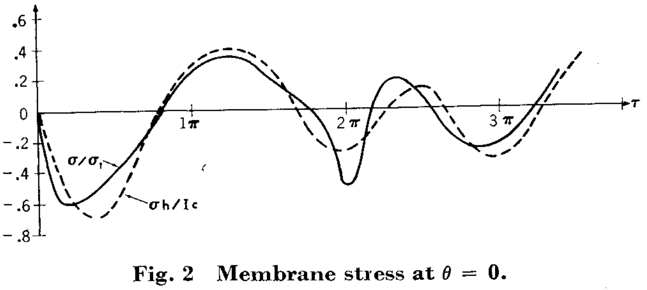


**Fig. 2 Membrane stress at $\theta = 0$.**

With this time shift there is close agreement between both response curves. Similar comparisons of these response data for $\theta = \pm\pi/2$ and $\theta = \pi$ with the same time shift for the moving load response show similar agreement and are available from the authors. The total impulse imparted to the cylinder for each loading in Fig. 2 is equal, and $\alpha = 0.73$ corresponds to the ratio of the bar velocity of a steel ring to the detonation velocity of the sheet explosive EL-506D.

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# Shock Slip Analysis of Merged Layer Stagnation Point Air Ionization

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### Introduction

ANALYTICAL analysis of high-speed, high-altitude flow about a blunt body is complicated by the presence of chemical nonequilibrium phenomena in conjunction with rarefaction effects, which creates a chemically reacting, fully viscous, merged shock-layer flowfield. Most prior investigations of this problem (e.g., Refs. 1 and 2) have made the assumption that the species conservation equations can be uncoupled from the fluid-dynamic conservation equations so that previously obtained frozen-flow solutions can be used to perform the nonequilibrium species composition calculations. A recent paper by Dellinger[3] suggested that such an approximate calculation may not be strictly correct for prediction of merged-layer ionization. In addition, various assumptions regarding the chemical and diffusion models often are made with little or no attention given to their effect on the resultant solution. The present Note is an attempt to clarify the effects of reaction rates, species diffusion, and coupled vs decoupled species concentration calculations on merged-layer air ionization predictions using a nonequilibrium thin viscous shock-layer analysis for comparison with the results of Refs. 1–3.

### Analysis

The reader is referred to Refs. 4 and 5 for complete details of the analytical method used in the present work. Basically,

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